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MOTION OF A HYDROFOIL ABOVE AN INTERFACE BETWEEN TWO HEAVY FLUIDS

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The problem of motion of a hydrofoil near the interface between two media is primarily interesting owing to practical applications. The main results in this area were obtained by solving the problem of hydrofoil motion under the interface between two heavy liquids, i.e., in a denser medium [1]. However, it is of interest to consider the hydrofoil motion in a less dense liquid, in particular, in air above water. This is the subject of the present study.

We constructed an algorithm for solving this problem which allows us to perform highly accurate calculations. For a hydrofoil moving above the air–water interface, we analyzed the dependence of the distributed and total hydrodynamic characteristics and also the interface shape on the problem parameters.

1. Let us consider the linear boundary-value problem of steady-state motion of a hydrofoil L above the interface between two media. The liquid is assumed to be ideal, incompressible, heavy, and homogeneous in layers D_1 and D_2. We introduce a coordinate system Oxy with the Ox axis directed along the unperturbed interface and with the Oy axis passing through the leading edge of the hydrofoil (Fig. 1). The following notation is used: g is the acceleration of gravity; ρ_k and V_k are the density and velocity at infinity in front of the hydrofoil in layer D_k ($k = 1$ and 2); H is the distance between the leading hydrofoil edge and the unperturbed medium interface; and α and b are the angle of attack and the hydrofoil chord, respectively.

To describe the motion of a liquid in layer D_k, we introduce the complex velocities $\overline{V}_k(z)$ ($k = 1$ and 2), $z = x + iy$. For the function $\overline{V}_k(z)$ the following conditions must be satisfied: the analyticity in D_k (beyond the contour L for $k = 2$), the continuity of both pressure and the normal velocity component in passage through the interface between two media, the decay of perturbed velocities at infinity in front of the hydrofoil, and the absence of a liquid flow through the contour L. In addition, we search for a solution in the class of functions that satisfy the Joukowski postulate in the trailing edge of the hydrofoil.

This boundary-value problem can be reduced to either of two integral equations:

$$\operatorname{Im}\{\overline{V}_0(z)\,e^{i\theta(s)}\} = 0, \quad z \in L; \tag{1.1}$$

$$-\frac{1}{2}\,\gamma(s) = \operatorname{Re}\{\overline{V}_0(z)\,e^{i\theta(s)}\}, \quad z \in L. \tag{1.2}$$

Here s is the arc coordinate of point $z \in L$; $\gamma(s)$ is the intensity of the vortex layer simulating L; $\theta(s)$ is the angle between the tangent to L at point $z(s)$ and the Ox axis; $\overline{V}_0(z) = \overline{V}_2(z)$ for $z \in L$, and the complex velocities $\overline{V}_k(z)$ ($k = 1$ and 2) are determined from the formulas

$$\overline{V}_k(z) = V_{k\infty} + \frac{1}{2\pi i}\int\limits_L K_k(z,\zeta)\,\gamma(s)\,e^{-i\theta(s)}\,d\zeta, \quad k = 1, 2; \tag{1.3}$$

$$K_1(z,\zeta) = \frac{V_{1\infty}}{V_{2\infty}}\left\{\frac{m_{12}^2}{\pi i}\,\frac{1}{z-\zeta} - \frac{\nu_1 m_{12}^2}{\pi}\int\limits_0^\infty \frac{e^{-i\lambda(z-\zeta)}}{\lambda-\nu_1}\,d\lambda - \nu_1 m_{12}^2 i\,e^{-i\nu_1(z-\zeta)}\right\}; \tag{1.4}$$

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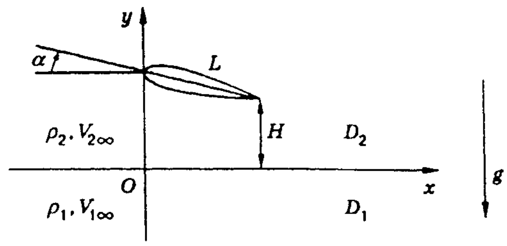

Fig. 1

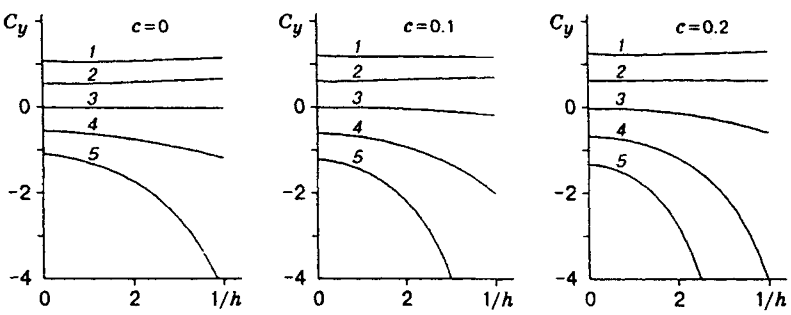

Fig. 2

$$K_2(z,\zeta)=\frac{1}{2\pi i}\frac{1}{z-\zeta}-\frac{m_{12}}{2\pi i}\frac{1}{z-\bar\zeta}-\frac{\nu_1 m_{12}^2}{\pi}\int\limits_0^\infty\frac{e^{i\lambda(z-\bar\zeta)}}{\lambda-\nu_1}\,d\lambda-\nu_1 m_{12}^2 i\,e^{i\nu_1(z-\bar\zeta)}, \qquad (1.5)$$

where

$$m_{12}^1=\frac{\rho_1 V_{1\infty}^2}{\rho_1 V_{1\infty}^2+\rho_2 V_{2\infty}^2};\quad \nu_1=\frac{g(\rho_1-\rho_2)}{\rho_1 V_{1\infty}^2+\rho_2 V_{2\infty}^2};\quad m_{12}=m_{12}^1-m_{12}^2;\quad m_{12}^2=\frac{\rho_2 V_{2\infty}^2}{\rho_1 V_{1\infty}^2+\rho_2 V_{2\infty}^2}.$$

Expressions (1.4) and (1.5), derived for $K_k(z,\zeta)$ ($k=1$ and 2) using the method proposed in [2], are exact solutions of the corresponding boundary-value problem of a vortex of unit intensity.

According to [3], a system of integral equations that do not degenerate in the limiting case of an infinitely small hydrofoil thickness was derived from (1.1) and (1.2). A method for solving this system in the class of functions $\gamma(s)$ satisfying the Joukowski postulate is presented in [4]. The interface shape is given by

$$f(x)=-\frac{1}{\nu_1}\mathrm{Re}\Big\{m_{12}^1\Big(\frac{\overline{V_1(z)}}{V_{1\infty}}-1\Big)-m_{12}^2\Big(\frac{\overline{V_2(z)}}{V_{2\infty}}-1\Big)\Big\},\quad z=x,$$

where $V_k(z)$ ($k=1$ and 2) is determined by formulas (1.3)–(1.5).

The pressure distribution over the hydrofoil, the total hydrodynamic forces R_x and R_y, and also the moment M of hydrodynamic forces are calculated as described in [5].

2. Calculations were performed for a symmetric Joukowski profile. The calculation algorithm was tested, using the well-known solutions of the problems of infinite fluid flow past a Joukowski hydrofoil and motion of the hydrofoil above a solid flat screen [4]. In this case, the relative calculation error was not greater than 1%.

The dimensionless parameters of the problem are the Froude number $\mathrm{Fr}=V_{2\infty}/\sqrt{gb}$, the ratio of densities $\rho_*=\rho_2/\rho_1$, the ratio of flow velocities $v_*=V_{2\infty}/V_{1\infty}$, the dimensionless distance between the trailing edge and the unperturbed interface $h=H/b$, and the relative hydrofoil thickness c.

We calculated the standard aerodynamic coefficients C_x, C_y, and C_m, which determine the total aerodynamic forces R_x and R_y, and the moment M relative to the leading edge of the airfoil, the dimensionless distance between the pressure center and the leading edge $C_d=C_m/(C_y\cos\alpha-C_x\sin\alpha)$, and the pressure distribution over the hydrofoil contour, i.e., the coefficient $C_p=2(p-p_\infty)/(\rho_2 V_{2\infty}^2)$, where p and p_∞ are the

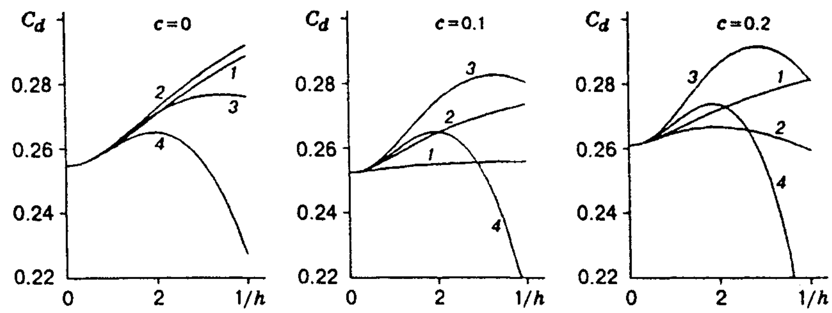

Fig. 3

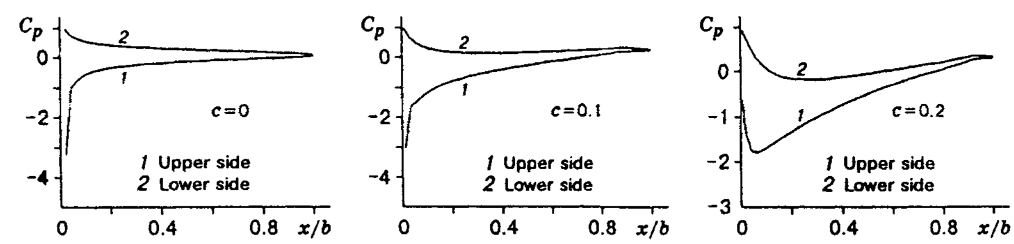

Fig. 4

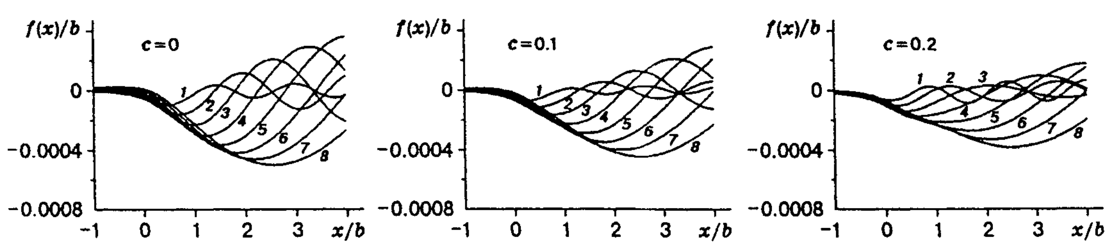

Fig. 5

hydrodynamic pressures at a given and infinitely distant points.

For the problem of motion of an airfoil above the air–water interface ($\rho_* = 0.00125$ and $v_* = 1$), a numerical experiment was carried out to estimate the influence of these parameters on the distributed and total hydrodynamic hydrofoil characteristics and on the interface shape. The main results are shown in Figs. 2–5.

Figure 2 shows the curve of C_y versus the parameter h for $c = 0$, 0.1, and 0.2; $\alpha = 10$, 5, 0, -5, and -10^0 (curves 1–5); and $\text{Fr} = 1$. As the hydrofoil thickness increases and the distance between the hydrofoil and the interface decreases, the lift modulus increases. This effect is especially noticeable for the negative angles of attack. A similar character of the effect of the hydrofoil thickness was observed for the moment coefficient C_m. In this case, the coefficient C_x is of the order of 10^{-4}. Calculations show that there are no noticeable changes in the coefficients C_x, C_y, and C_m with an increase of the Froude number.

Figure 3 demonstrates the location of the pressure center on the hydrofoil versus the parameter h for $\alpha = 10$, 5, -5, and -10^0 (curves 1–4) and $c = 0$, 0.1, and 0.2 for $\text{Fr} = 1$. The behavior of the coefficient C_d appeared to be similar to that for the problem of motion of a hydrofoil above a screen [4].

Of interest is the pressure distribution over the hydrofoil contour (Fig. 4). We chose the following parameters: $c = 0$, 0.1, and 0.2; $\alpha = 5^0$, $h = 0.5$, and $\text{Fr} = 1$. A substantial change in the pressure distribution is observed with increasing hydrofoil thickness.

The interface shape versus the Froude number (curves 1–8 correspond to $\text{Fr} = 0.5$, 0.6, 0.7, 0.8, 0.9, 1.0, 1.1, and 1.2) for $h = 0.5$ and $\alpha = 5°$ for $c = 0$, 0.1, and 0.2, is shown in Fig. 5. In this case, a considerable effect of the hydrofoil thickness and Froude number is observed.

Numerical results enable the following conclusions to be drawn. The distributed and total hydrodynamic characteristics of a hydrofoil moving above the air–water interface are practically independent of the Froude number. This allows this interface to be simulated by a screen in the calculations of hydrodynamic reactions on the hydrofoil. However, the shape of the air–water interface depends significantly on the Froude number.

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